5. V. V. Kafarov, Methods of Chemical Cybernetics in Chemistry and Chemical Engineering [in Russian], Khimiya, Moscow (1971).
6. A. Yu. Zakgeim, Introduction to Modeling of Chemical-Engineering Processes [in Russian], Khimiya, Moscow (1973).
7. S. Middleman, Flow of Polymers [Russian translation], Mir, Moscow (1971).
8. Ya. Didushinskii, Principles of the Design of Catalytic Reactors [in Russian], Khimiya, Moscow (1972).
9. Chemist's Handbook [in Russian], Vol. 3, Khimiya, Moscow—Leningrad (1964).
10. N. V. Tyabin, Rheological Cybernetics, Volgograd Polytechnic Institute, Volgograd (1978).

ONE APPROACH TO CALCULATING A TURBULENT BOUNDARY LAYER ON A SURFACE WITH A COMPLIANT COATING

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A method is proposed for calculating a turbulent boundary layer on a surface with a viscoelastic coating. The method is based on the introduction of the van Driest damping function to account for the effect of the coating on the boundary layer.

It is now considered proven that the application of a layer of viscoelastic (elastic, compliant) material to the surface of a body moving in a liquid or gas may lead to a 50-60% reduction in the drag associated with the body. This has been confirmed by several experiments with different types of coatings (we may recommend the survey [1], which contains an extensive bibliography). At the same time, there are studies in which the investigators not only failed to find a reduction in drag, but in fact observed the reverse effect.

Theoretical study of a turbulent boundary layer on such surfaces is complicated not only by transient boundary conditions, but also by a lack of detailed knowledge of the dynamics of viscoelastic materials. Only in the most recent works [2, 3] have attempts been made to take a combined approach to this problem.

Friction on a solid surface is limited by the interaction of turbulent and viscous transfer near the surface. One effective approach to accounting for the interaction of viscous and turbulent transfer in a turbulent boundary layer close to a solid surface is the introduction of so-called damping functions, reflecting the dynamics of pulsations in a viscous fluid. Such functions have been obtained by different methods by Loitsyanskii, Vulis, and van Driest [4] for a boundary layer on a flat plate. For example, van Driest proposed a structural form of damping function for the length of the displacement path l in a plane boundary layer on the basis of an analysis of harmonic oscillations of an infinite flat plate in an unbounded incompressible viscous fluid (the Stokes problem) decaying as they penetrate into the fluid according to the law $\exp[-y(\omega/\nu)^{1/2}]$, where ω is the frequency of the oscillations; ν is the kinematic viscosity coefficient of the liquid:

$$l^+ = \varkappa y^+ F(y^+),\ F(y^+) = 1 - \exp(-y^+/a), \tag{1}$$

where $y^+ = yu_*/\nu$; $\varkappa = 0.4$; $a = 26$ are universal empirical constants.

Further, several authors used van Driest's idea as a basis for constructing damping functions for certain more complex flows (turbulent boundary layer on porous flat [5] and cylindrical [6] surfaces).

The attractiveness of van Driest's idea derives first of all from the fact that well-known solutions of the Navier—Stokes equations are used (in one form or another) to construct a structural form of damping function for a given complex turbulent flow. In such a situation, it would be of interest to construct a damping function of the van Driest type which would convey information on the properties of the viscoelastic coating mentioned earlier.

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This would make it possible to reduce the problem of calculating the turbulent boundary layer to using well-known methods (see [4]), with almost no change in the algorithms of the latter.

The present work proposes the construction of a structural form of damping function on the basis of an analysis of natural oscillations of a liquid next to a well in the case where the oscillations are related to the propagation of traveling-wave-type perturbations over the surface of a layer of viscoelastic material (on a flat hard plate).

1. We examine the motion of a viscous incompressible liquid next to an infinite flat plate covered by a layer of viscoelastic isotropic material of constant thickness h.

The motion of the liquid is described by the Stokes–Navier equations:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\Delta u, \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\Delta v, \tag{3}$$

the continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4}$$

and a system of boundary conditions in the outer flow and conditions of adhesion of the coating to the surface:

$$u(x,\ y=\infty,\ t)=U_\infty, \tag{5}$$

$$u(x,\ y=y_c(t),\ t)=u_c, \tag{6}$$

$$v(x,\ y=y_c(t),\ t)=v_c, \tag{7}$$

where $y_c(x, t)$, u_c, v_c are unknown perturbations of the form of the coating and values of velocity components of the coating, which must be determined in accordance with the equations of motion of the liquid and viscoelastic medium and the kinematic relation.

$$v_c=\frac{\partial y_c}{\partial t}+u_c\frac{\partial y_c}{\partial x}. \tag{8}$$

Let us represent the sought quantities u, v, and p in the form of sums

$$u=u^\circ+u^1,\ v=v^\circ+v^1,\ p=p^\circ+p^1, \tag{9}$$

the first terms of which represent components of velocity and pressure next to the hard plate. The second terms represent complements connected with the presence of the viscoelastic coating.

It is clear from physical considerations that the magnitude of the surface perturbation y_c should be small (less than the thickness of the viscous sublayer): $y_c = \varepsilon f(x, t)$, $\varepsilon \ll 1$, so that naturally we should try to find a solution of the problem in the form of expansions in the small parameter ε:

$$u^1=\varepsilon u_1^1+\varepsilon^2 u_2^1+\ldots,\ v^1=\ldots,\ p^1=\ldots \tag{10}$$

As might be expected, substitution of (9) and the expansions (10) into (2), (3) leads in a first approximation to the Stokes–Navier equations for flow next to a solid boundary; in a second approximation it leads to the linear problem of perturbed motion. Similarly, we may obtain the boundary conditions for perturbed motion:

$$u_1^1(x,\ \infty,\ t)=0,\ v_1^1(x,\ \infty,\ t)=0,$$
$$u_1^1(x,\ 0,\ t)=-f\left(\frac{\partial u^\circ}{\partial y}\right)_{y=0}=-\frac{\tau^\circ}{\rho\nu}f\equiv-\frac{u_*^2}{\nu}f, \tag{11}$$

where τ° is the shear stress on the hard plate; u_* is the dynamic velocity. In deriving the boundary conditions at y = 0, it was assumed that the viscoelastic coating does not undergo displacements along the x axis ($u_c = 0$), since experimental studies [7] show that the magnitude of shear stress pulsations on a wall in a turbulent boundary layer are at least one order lower than the magnitude of the pressure pulsations.

Direct solution of a system of perturbation equations together with transient boundary conditions is a problem which is unjustifiably complex for the purpose of obtaining a "structural" type solution. In connection with this, it is expedient to make certain simplifying assumptions. For example, it may be assumed that the pressure pulsation on a viscoelastic surface is the same as on a rigid surface: $p_1^1(x, 0, t) = p^{\circ'}(x, 0, t)$.

It was shown in an experiment in [8] that the wave-frequency spectrum of the pressure pulsations in a turbulent boundary layer has a fairly distinct maximum and that the mode corresponding to this maximum can be represented in the form

$$\overset{\circ'}{p_w} = p_0 \exp(i\lambda x - i\Omega t),$$

where $\lambda \approx 0.42/\delta^*$; $\Omega \approx 0.35/\delta^*$; δ^* is the displacement thickness; p_0 is a characteristic measure of the pressure pulsations in the boundary layer, such as the standard deviation of the pressure on the wall $p_0^2 = [\langle p^{\circ'})^2 \rangle]_{y=0}$.

It should be noted that our selection of a single traveling wave as the form of the pressure perturbation does not disturb the generality of the formulation by virtue of the linearity of the problem being examined; the perturbation may be represented, e.g., by a train of traveling waves with an appropriate frequency spectrum. The latter case would in fact more accurately reflect the actual situation in a turbulent boundary layer.

For a high-frequency perturbation such as the one being examined, the inertial terms in the equations of perturbed motion may be neglected in relation to the transient and viscous terms. Thus, the equations of the approximation under discussion will have the form

$$\frac{\partial u_1^1}{\partial t} = -\frac{1}{\rho}\frac{\partial p_1^1}{\partial x} + \nu\Delta u_1^1,$$
$$\frac{\partial v_1^1}{\partial t} = -\frac{1}{\rho}\frac{\partial p_1^1}{\partial y} + \nu\Delta v_1^1,$$
$$\frac{\partial u_1^1}{\partial x} + \frac{\partial v_1^1}{\partial y} = 0. \tag{12}$$

Differentiating the first of these equations with respect to x and the second with respect to y and adding, we can obtain the equation for pressure

$$\Delta p_1^1(x, y, t) = 0, \tag{13}$$

the boundary condition for which is

$$p_1^1(x, 0, t) = p_0 \exp(i\lambda x - i\Omega t). \tag{14}$$

Equations (13), (14) constitute the well-known Dirichlet problem for an upper half plane and have the exact solution:

$$p_1^1(x, y, t) = p_0 \exp(i\lambda x - i\Omega t - \lambda y).$$

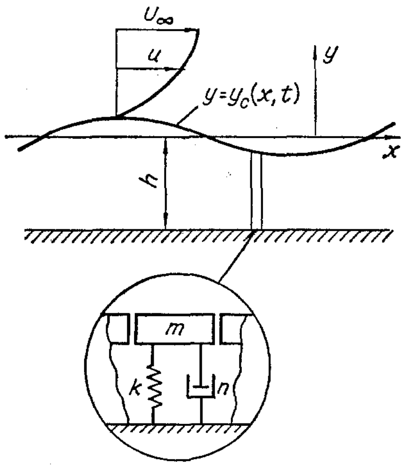
2. A traveling wave from a pressure perturbation causes a corresponding, as-yet-unknown perturbation of the form of the boundary $y_c(x, t)$ so that the boundary conditions (11) are indeterminate at this stage. As an equation for y_c, we propose to use an equation satisfying one of the simplest models of a viscoelastic medium and qualitatively describing the property of elastic aftereffect — the Kelvin—Foight model [9]. In accordance with this model, the medium is a set of elements including a Hookean solid (characteristic modulus of elasticity k) and a damper (characteristic viscosity coefficient n) (Fig. 1).

Thus, the equation of wall deformation according to Kelvin—Foight will have the form

$$m\frac{\partial^2 f}{\partial t^2} + n\frac{\partial f}{\partial t} + kf = p_0 \exp(i\lambda x - i\Omega t), \tag{15}$$

where $m = \rho_c\Delta_s$ is the effective mass of a unit area of the coating; Δ_s, effective thickness of the oscillating layer, for which we will take the thickness of the Stokes layer $\Delta_s = (n/\rho_c\Omega)^{1/2}$ if $\Delta_s < h$ and $\Delta_s = h$ if $(n/\rho_c\Omega^{1/2} > h)$.

Given the above formulation, it is to be expected that the surface pressure perturbation induced by the traveling wave will have the same form as the wave, with the same parameters: $f(x, t) = Y\exp(i\lambda x - i\Omega t)$, $Y = Y_r + iY_i$. Meanwhile, the amplitude Y should be determined from (15). Substitution of $f(x, t)$ in (15) gives us



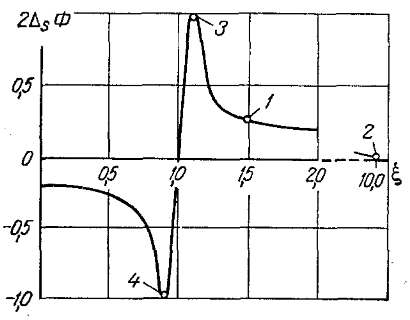


Fig. 1 Fig. 2

Fig. 1. Diagram of flow and Kelvin–Foight model of viscoelastic coating.

Fig. 2. Curve of the function $2\Delta_s\Phi(\xi)$. $\Delta_s=0.1$.

$$Y_r = A\frac{\xi^2-1}{(\xi^2-1)^2+D^2\xi^2},\quad Y_i = -A\frac{D\xi}{(\xi^2-1)^2+D^2\xi^2},$$

where $A=p_0/m\Omega^2$; $\Omega_0^2=k/m$ is the natural vibration frequency of the coating; $\xi = \Omega_0/\Omega$; $D = n/m\Omega_0$ is the damping factor, $D = \Delta_s/\xi$.

3. In the dimensionless variables $\bar{x} = x/L$, $\bar{y} = y/\delta_s$, $\bar{t} = \Omega t$, $\bar{u} = u/U_\infty$, $\bar{p} = p/p_0$, the equation for u from (12) takes the form (bars omitted)

$$\frac{\partial u}{\partial t} = -\alpha\frac{\partial p}{\partial x}+\beta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}. \tag{16}$$

Here $L = 1/\lambda$ is the wavelength of the surface perturbation; $\delta_s = (\nu/\Omega)^{1/2}$, Stokes scale in the liquid; $p\equiv p_1^1$; $u\equiv u_1^1$; $\alpha=p_0/\rho U_\infty\Omega L$; $\beta=\delta_s/L\approx 0.5\mathrm{Re}^*$; Re*, Reynolds number, determined from the displacement thickness. A solution of (16) satisfying boundary condition (10) and finite at $y = +\infty$ has the form

$$u(x,\ y,\ t) = \alpha Y\exp(ix-it-\beta y)-\left(\frac{u_*^2}{\nu U_\infty^2}+\alpha\right)Y\exp[(\Lambda_r-\beta)y+i\Lambda_i y+ix-it],$$

where

$$\Lambda_r = \beta-\frac{1}{2}\left(\sqrt{\sqrt{1+\beta^4}+1}+\sqrt{\sqrt{1+\beta^4}-1}\right).$$

As can be seen, the solution consists of two traveling waves with an amplitude which decays along y, the amplitude of the second wave here decaying more rapidly ($\beta-\Lambda_r > \beta > 0$).

It can thus be proven that the character of pulsation damping is determined by the law of decay of the amplitude of the second wave.

4. By virtue of the fact that the perturbations introduced into the elastic surface are small, we may represent the damping function in the form of the sum of the functions for the base and perturbed flows:

$$F(y^+) = 1-\exp(-y^+/a)-[1-\exp(-\theta\Lambda_r y^+)]\left(\frac{u_*^2}{\nu U_\infty}+\frac{p_0\lambda}{\rho\Omega}\right)\qquad\frac{p_0}{\rho_c\Delta_S\Omega^2 U_\infty}\Phi(\xi),$$

where $\Phi(\xi)=(\xi^2-1)/[(\xi^2-1)^2+\Delta_S^2]$ (Fig. 2), $y^+=yu_*/\nu$; $\theta=(\nu\Omega)^{1/2}/u_*$. Here, the first two terms represent the normal van Driest function for a boundary layer on a hard surface (1): ΔF is a complement connected with the presence of the viscoelastic surface and vanishing as the coating degenerates into a rigid surface ($k\to\infty$).

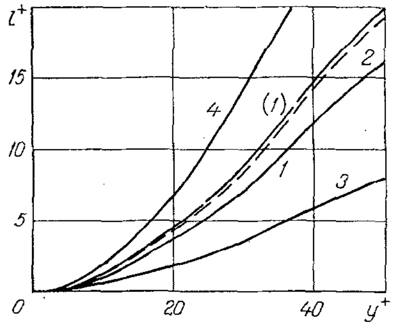


Fig. 3. Length of displacement path for the case of a hard plate [van Driest formula (1)] and four variants of coating.

The form of $F(y^+)$ shows that the effect of the coating parameters on the length of the displacement path $l^+(y^+)=\varkappa y^+F(y^+)$ and, thus, on turbulent friction is ambiguous in character and is determined by the interaction of the natural vibration frequency of the coating Ω_0 and the characteristic frequency of the pressure pulsations. Consequently, the effect will always be positive (friction will decrease) when $\xi>1$ $(\Omega_0>\Omega)$ and negative in the opposite case. The magnitude of the effect is determined by all of the parameters of the problem, and it is not difficult to see that there exists a relationship between these parameters for which the effect will be maximal:

$$\left(\frac{\Omega_0}{\Omega}\right)^2=1+\left(\frac{n}{\rho_c\Omega}\right)^{1/2}.$$

Thus, even a qualitative analysis of the proposed method illuminates the potential for obtaining conflicting results in different experiments, since the intuitive nature of the selection of a coating material and imprecise and incomplete empirical determination of its characteristics can guarantee the selection of a successful combination of coating and flow parameters only with a specified probability.

Approximate numerical results are shown in Fig. 3 for the case of a flow of air (U = 40 m/sec, p_0 = 4 N/m², $u^*\approx$ 0.5 m/sec) over four types of coatings: 1) polyurethane foam ($E = 2\cdot10^4$ N/m², ρ_0 = 30 kg/m³, n/ρ_c = 30 kgf/m²) of 0.07 m thickness (point 1 on curve 1 in Fig. 2); 2) same material but of 0.01 m thickness (point 1 on curve 2 in Fig. 2); 3) molded polyurethane foam ($E = 8\cdot10^3$ N/m², $\rho_c = 10^2$ kg/m³) of 0.03 m thickness (point 3 on curve 3 in Fig. 2); 4) conventional heated polyurethane foam ($n_T = 0.8\, n_{T_0}$)* (point 4 on curve 4 in Fig. 2).

Apart from the basic possibilities of the method, the path of the curves demonstrates the effect of such parameters as the thickness of the coating — chosen in experiments with regard for processing considerations — or temperature — which is generally not controlled. As can be seen, the effect may prove to be positive, ambiguous, or adverse.

NOTATION

x, y, Cartesian coordinates; u, v, velocity components along the axes x, y; p, pressure; m, n k, parameters of the Kelvin—Foight model representing the density, viscosity, and elasticity of the coating material; F, van Driest damping function; Δ, Laplace operator.

LITERATURE CITED

1. M. G. Fisher, L. M. Weinstein, D. M. Bushnell, and R. L. Ash, "Compliant wall-turbulent skin friction reduction research," AIAA Paper, No. 75-833, 1-30 (1975).
2. D. T. Tsahalis, "On the theory of skin friction reduction by compliant walls," AIAA Paper, No. 686, 1-11 (1977).

*Most polymers are characterized by a substantial reduction in viscosity properties within a narrow temperature range T = 10–30°C [1].

3. G. A. Voropaev, "Calculation of certain characteristics of a turbulent boundary layer on a compliant surface," in: Mathematical Methods of Studying Flow Hydrodynamics [in Russian], Naukova Dumka, Kiev (1978), pp. 9-13.
4. K. K. Fedyaevskii, A. S. Ginevskii, and A. V. Kolesnikov, Calculation of the Turbulent Boundary Layer of an Incompressible Fluid [in Russian], Sudostroenie, Leningrad (1973).
5. W. B. Nicoll, A. B. Strong, and K. A. Woolner, "On the laminar motion of a fluid near an oscillating porous infinite plane," Trans. ASME, Ser. E, 35, No. 1, 324-327 (1968).
6. A. B. Airapetov, "Laminar motion of a viscous incompressible fluid next to a permeable cylinder vibrating in the fluid," Uch. Zap. TsAGI, 4, No. 5, 88-92 (1973).
7. K. R. Sreenivasan and R. A. Antonia, "Properties of wall shear stress fluctuations in a turbulent duct flow," Trans. ASME, Ser. E, 44, No. 3, 389-395 (1977).
8. B. M. Efimtsov and S. E. Shubin, "Probability characteristics of pressure pulsations in a boundary layer on the surface of an aircraft," in: Aeronautical Acoustics, Tr. TsAGI, No. 1655 (1975), pp. 3-14.
9. P. M. Ogivalov, N. I. Malinin, V. P. Netrebenko, and B. P. Kishkin, Structural Polymers [in Russian], Vol. 1, Moscow State Univ. (1972).

NUMERICAL SOLUTION OF A TWO-DIMENSIONAL PROBLEM OF THE TRANSIENT HYDRODYNAMICS OF A COMPRESSIBLE NON-NEWTONIAN FLUID

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A solution is presented for a transient two-dimensional problem of the hydrodynamics of a compressible non-Newtonian fluid connected with the propagation and damping of shock waves in a well.

The physical processes connected with well drilling have been studied in increasing detail in recent times, a fact related to the seriousness of the consequences of emergency situations at oil and gas extraction sites. Theoretical study of the hydrodynamics of wells is complicated by the need to solve problems for non-Newtonian fluids — which includes most drilling fluids. Here, most of the work that has been done has examined unidimensional and quasi-unidimensional hydrodynamic problems, with investigators neglecting or averaging two-dimensional and nonlinear effects [1, 2].

Examined below is a two-dimensional transient problem of the hydrodynamics of a compressible non-Newtonian fluid with allowance for several nonlinear phenomena which might exert a marked effect in the generation and propagation of shock waves in long channels. We chose for the form of the rheological equation a relation describing shear stress as an exponential function of shear rate, which is a good approximation for most drilling fluids.

Given this model, we may study a whole range of problems of dynamics connected with the opening up of beds with pressure anomalies, the closing of pipe connections, start-up of pumps, lowering and raising of drilling equipment, etc. Here, we examine the first of these problems and solve it by the method of fractional steps [3] on an R-1040 computer.

The unsteady motion of a non-Newtonian fluid is described by the following dynamic equation [4, 5]:

$$\rho\frac{\partial \mathbf{v}}{\partial t} + \rho\nabla\mathbf{v}\cdot\mathbf{v} = -\nabla P + \operatorname{div}\bar{\tau}' + \rho\mathbf{g}. \tag{1}$$

In the case of a compressible fluid, apart from the shear stresses, the viscous stress tensor should also account for linear strain [4], i.e.,

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